

**GCE Examinations**  
**Advanced Subsidiary / Advanced Level**  
**Further Pure Mathematics**  
**Module FP2**

**Paper C**

**MARKING GUIDE**

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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## FP2 Paper C – Marking Guide

1. 
$$\begin{aligned}\rho &= \frac{ds}{d\psi} = 12 \sec^2 \psi \times \sec \psi \tan \psi && \text{M1 A1} \\ &= 12 \sec^3 \psi \tan \psi && \text{A1} \\ \psi &= \frac{\pi}{4}, \rho = 12(\sqrt{2})^3(1) = 24\sqrt{2} && \text{M1 A1 (5)}\end{aligned}$$

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2. 
$$\begin{aligned}\frac{5 \cosh x}{\sinh x} + 1 &= \frac{7}{\sinh x} && \text{M1} \\ 5 \cosh x + \sinh x &= 7 && \text{A1} \\ \frac{5}{2}(e^x + e^{-x}) + \frac{1}{2}(e^x - e^{-x}) &= 7 && \text{M1} \\ 3e^x + 2e^{-x} &= 7 && \\ 3e^{2x} - 7e^x + 2 &= 0 && \text{A1} \\ (3e^x - 1)(e^x - 2) &= 0 && \text{M1} \\ e^x = \frac{1}{3} \text{ or } 2 &\therefore x = \ln \frac{1}{3} \text{ or } \ln 2 && \text{M1 A1 (7)}\end{aligned}$$

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3. (a) let  $y = \arccos x \therefore \cos y = x$   
 $\therefore -\sin y \frac{dy}{dx} = 1$  M1  
 $\frac{dy}{dx} = \frac{-1}{\sqrt{1-\cos^2 y}} = \frac{-1}{\sqrt{1-x^2}}$  M1 A1

(b)  $\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}} - \frac{1}{2} \frac{-2x}{1-x^2} = \frac{-1}{\sqrt{1-x^2}} + \frac{x}{1-x^2}$  M1 A1  
S.P.  $\therefore \frac{dy}{dx} = 0 \therefore \frac{x}{1-x^2} = \frac{1}{\sqrt{1-x^2}}$  M1  
 $x = \sqrt{1-x^2}$   
 $x^2 = 1 - x^2$  M1  
 $x^2 = \frac{1}{2}, 0 < x < 1, \therefore x = \frac{1}{\sqrt{2}}$  A1  
 $x = \frac{1}{\sqrt{2}}, y = \frac{\pi}{4} - \frac{1}{2} \ln \frac{1}{2} \therefore (\frac{1}{\sqrt{2}}, \frac{\pi}{4} - \frac{1}{2} \ln \frac{1}{2})$  M1 A1 (10)

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4. (a)  $3 - 6x - 9x^2 \equiv 3 - [(3x+1)^2 - 1]$   
 $\equiv 4 - (3x+1)^2 \therefore a = 4, b = 3, c = 1$

M1

A1

(b)  $\int \frac{1}{\sqrt{3-6x-9x^2}} dx = \int \frac{1}{\sqrt{4-(3x+1)^2}} dx$   
 $u = 3x+1, \frac{du}{dx} = 3$   
 $= \int \frac{1}{3} \frac{1}{\sqrt{4-u^2}} du$   
 $= \frac{1}{3} \arcsin\left(\frac{u}{2}\right) + c = \frac{1}{3} \arcsin\left(\frac{3x+1}{2}\right) + c$

M1

A1

M1 A1

(c)  $\int_{-\frac{1}{3}}^0 \frac{1}{3-6x-9x^2} dx = \int_{-\frac{1}{3}}^0 \frac{1}{4-(3x+1)^2} dx$   
 $u = 3x+1, \frac{du}{dx} = 3$   
 $= \int_0^1 \frac{1}{3} \frac{1}{4-u^2} du$   
 $= \frac{1}{3} \left[ \frac{1}{2} \operatorname{artanh}\left(\frac{u}{2}\right) \right]_0^1$   
 $= \frac{1}{6} [\operatorname{artanh}\frac{1}{2} - \operatorname{artanh}0]$   
 $= \frac{1}{6} \left[ \frac{1}{2} \ln\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right) - 0 \right] = \frac{1}{12} \ln 3$

M1

A1

M1 A1

(12)

5. (a)  $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

B1

(b) let  $y = \operatorname{artanh}\left(\frac{x^2-1}{x^2+1}\right) \therefore \tanh y = \frac{e^y - e^{-y}}{e^y + e^{-y}} = \frac{x^2-1}{x^2+1}$

 $(e^y - e^{-y})(x^2 + 1) = (e^y + e^{-y})(x^2 - 1)$ 
 $e^y[(x^2 + 1) - (x^2 - 1)] = e^{-y}[(x^2 - 1) + (x^2 + 1)]$ 
 $2e^y = 2x^2e^{-y}$ 
 $e^{2y} = x^2$ 
 $2y = \ln x^2 = 2 \ln x \therefore y = f(x) = \ln x$

M1

A1

M1

A1

(c)  $\int_1^{2e} \operatorname{artanh}\left(\frac{x^2-1}{x^2+1}\right) dx = \int_1^{2e} \ln x dx$   
 $u = \ln x, u' = \frac{1}{x}; v' = 1, v = x$   
 $I = [x \ln x]_1^{2e} - \int_1^{2e} x \times \frac{1}{x} dx$   
 $= [x \ln x - x]_1^{2e}$   
 $= 2e \ln(2e) - 2e - (\ln 1 - 1)$   
 $= 2e(\ln 2 + \ln e) - 2e + 1$   
 $= 2e \ln 2 + 2e - 2e + 1 = 2e \ln 2 + 1$

M1

A1

A1

M1

A1

(12)

6. (a)  $\frac{2x}{25} + \frac{2y}{9} \frac{dy}{dx} = 0 \therefore \frac{dy}{dx} = -\frac{9x}{25y}$  M1 A1
- at  $P$   $\frac{dy}{dx} = -\frac{9 \times 5 \cos \theta}{25 \times 3 \sin \theta} = -\frac{3 \cos \theta}{5 \sin \theta}$  M1
- $\therefore$  eqn. of normal is
- $y - 3 \sin \theta = \frac{5 \sin \theta}{3 \cos \theta} (x - 5 \cos \theta)$  M1 A1
- or  $5x \sin \theta - 3y \cos \theta = 16 \sin \theta \cos \theta$
- (b) at  $Q$ ,  $y = 0$ ,  $x = \frac{16}{5} \cos \theta \therefore Q(\frac{16}{5} \cos \theta, 0)$  M1 A1
- at  $R$ ,  $x = 0$ ,  $y = -\frac{16}{3} \sin \theta \therefore R(0, -\frac{16}{3} \sin \theta)$  M1 A1
- $\therefore S$  is  $(\frac{16}{5} \cos \theta, -\frac{16}{3} \sin \theta)$  M1
- of form  $(a \cos \theta, b \sin \theta) \therefore$  ellipse A1
- $\cos \theta = \frac{5}{16} x, \sin \theta = -\frac{3}{16} y$
- using  $\cos^2 \theta + \sin^2 \theta = 1$  gives  $\frac{25x^2}{256} + \frac{9y^2}{256} = 1$  M1 A1 (13)
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7. (a)  $u = \cos^{n-1} 2t, u' = 2(n-1) \cos^{n-2} 2t (-\sin 2t); v' = \cos 2t, v = \frac{1}{2} \sin 2t$  M1
- $I_n(x) = [\frac{1}{2} \cos^{n-1} 2t \sin 2t]_0^x - \int_0^x -(n-1) \cos^{n-2} 2t \sin^2 2t dt$  A1
- $I_n(x) = \frac{1}{2} \cos^{n-1} 2x \sin 2x - 0 + (n-1) \int_0^x \cos^{n-2} 2t (1 - \cos^2 2t) dt$  M1 A1
- $I_n(x) = \frac{1}{2} \cos^{n-1} 2x \sin 2x + (n-1) \int_0^x \cos^{n-2} 2t dt - (n-1) \int_0^x \cos^n 2t dt$  A1
- $I_n(x) = \frac{1}{2} \cos^{n-1} 2x \sin 2x + (n-1) I_{n-2}(x) - (n-1) I_n(x)$  M1
- $\therefore nI_n(x) = \frac{1}{2} \sin 2x \cos^{n-1} 2x + (n-1) I_{n-2}(x)$  A1
- (b)  $I_0(\frac{\pi}{4}) = \int_0^{\frac{\pi}{4}} dt = [t]_0^{\frac{\pi}{4}} = \frac{\pi}{4}$  M1 A1
- (c) area  $= \frac{1}{2} \int_0^{\frac{\pi}{4}} a^2 \cos^4 2\theta d\theta = \frac{1}{2} a^2 I_4(\frac{\pi}{4})$  M1 A1
- $nI_n(\frac{\pi}{4}) = \frac{1}{2} \sin \frac{\pi}{2} \cos^{n-1}(\frac{\pi}{2}) + (n-1) I_{n-2}(\frac{\pi}{4})$  M1
- $\therefore I_n(\frac{\pi}{4}) = \frac{n-1}{n} I_{n-2}(\frac{\pi}{4})$  A1
- $I_2(\frac{\pi}{4}) = \frac{1}{2} I_0(\frac{\pi}{4}) = \frac{\pi}{8}$  M1
- $I_4(\frac{\pi}{4}) = \frac{3}{4} I_2(\frac{\pi}{4}) = \frac{3\pi}{32}$  A1
- $\therefore$  area  $= \frac{1}{2} a^2 \times \frac{3\pi}{32} = \frac{3}{64} \pi a^2$  A1 (16)
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Total (75)

## **Performance Record – FP2 Paper C**